# Mathematical Modeling of Social Phenomena

Mixed bag/Epidemiology

# Class Layout

- Identification
- Preference aggregation
- Epidemiology
- Simulated epidemiology
- Markov processes

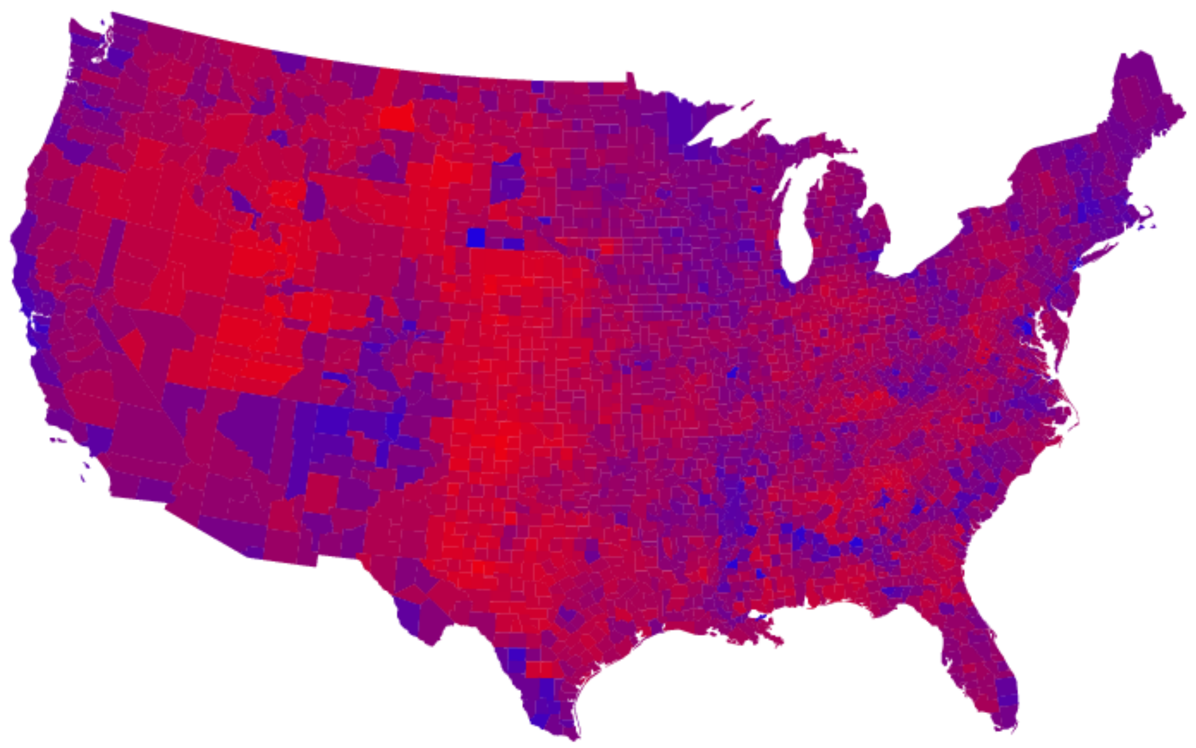
# Identification

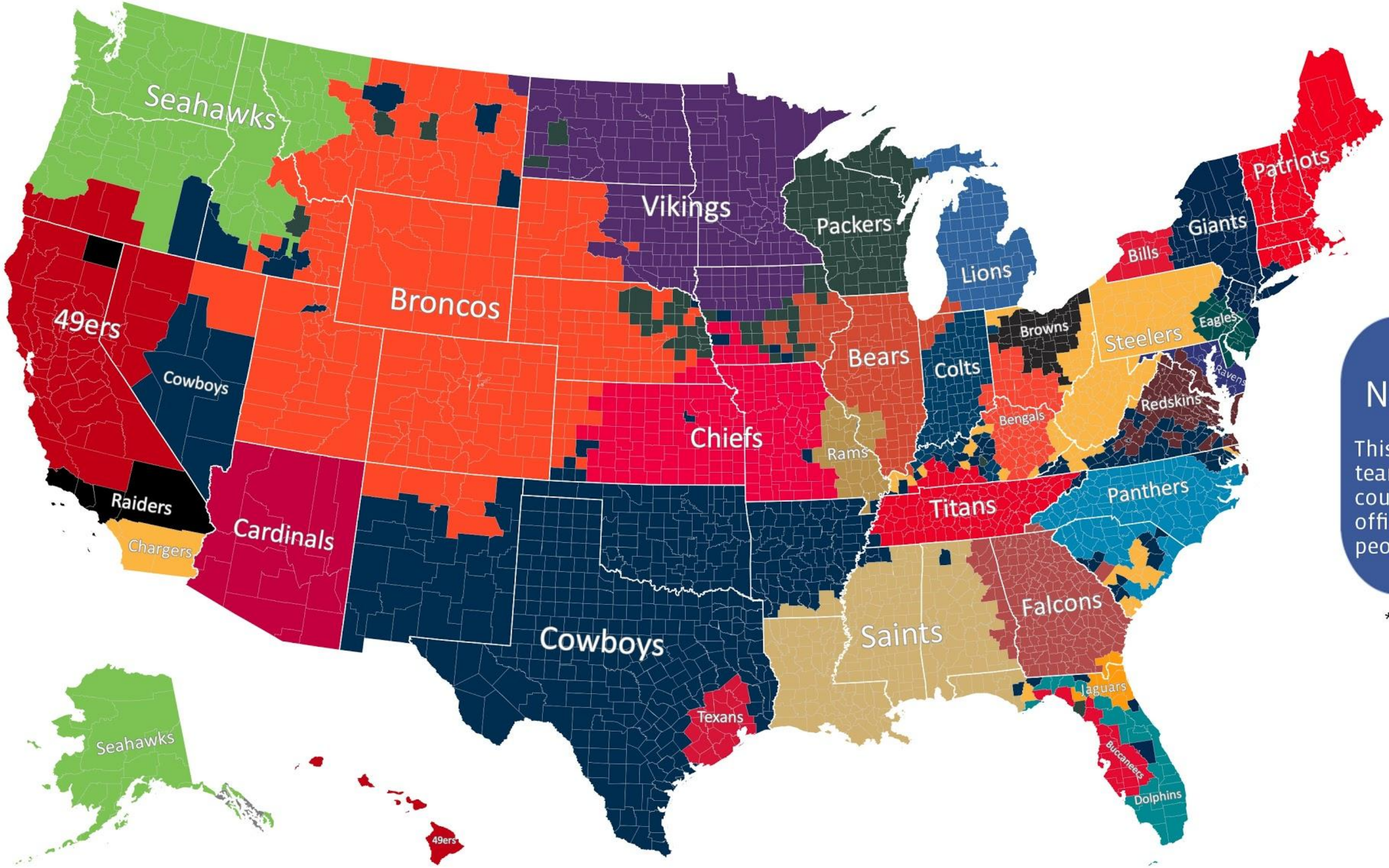
What is the better explanation?

# Identification - what explains?

Homophily

Peer effects



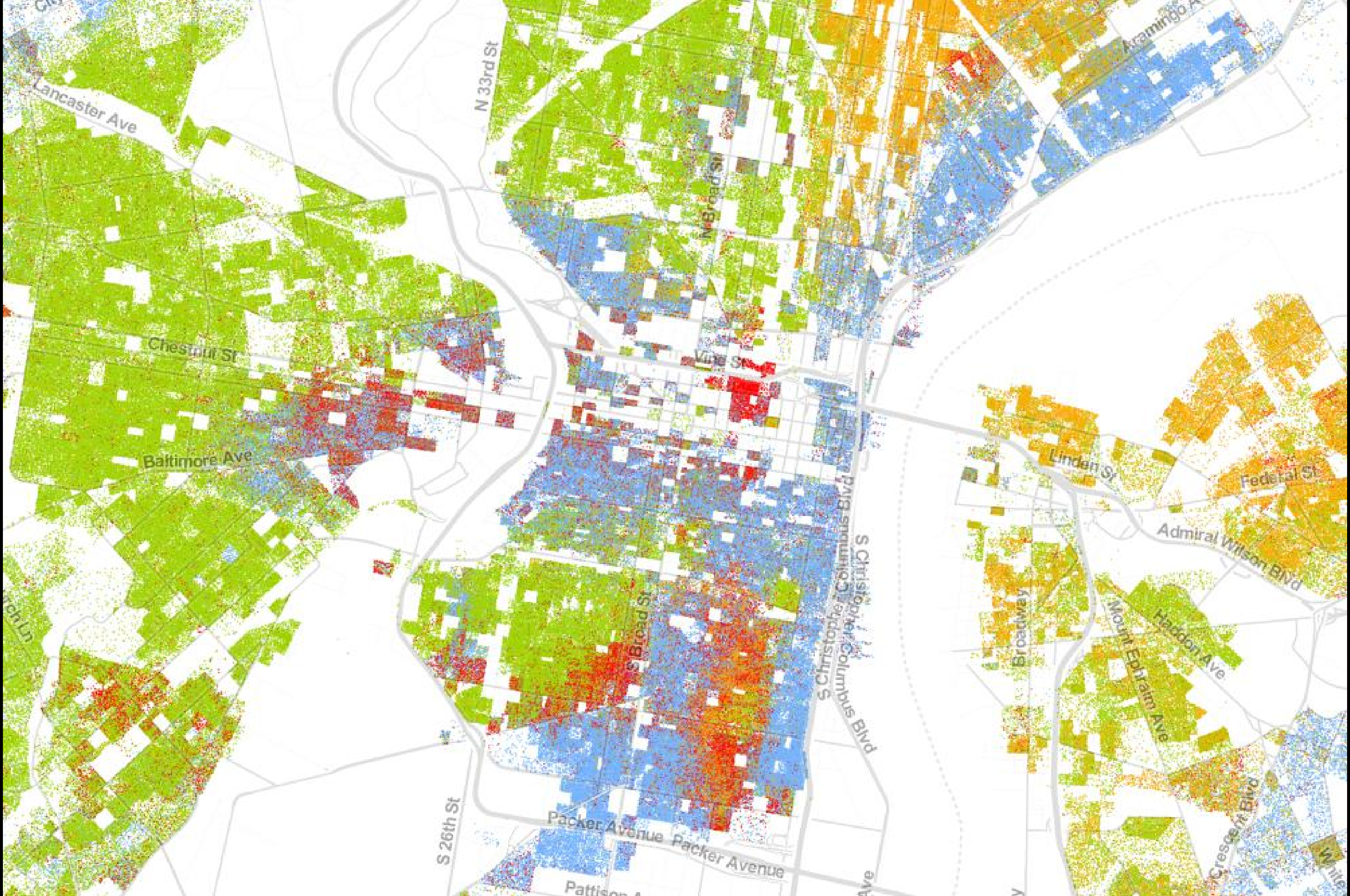



## Facebook Fandom Map 2014

# National Football League

This map displays Facebook fans of NFL teams across the United States. Each county is color-coded based on which official team page has the most "Likes" from people who live in that county.

* The New York Jets do not have a plurality of fans in any U.S. county.



Lancaster Ave
N 33rd St
N Broad St
Aramingo Ave
Chestnut St
Vine St
Baltimore Ave
Linden St
Federal St
Admiral Wilson Blvd
S Christopher Columbus Blvd
S Christopher Columbus Blvd
Broadway
Mount Ephraim Ave
Haddon Ave
S Broad St
S 26th St
Packer Avenue
Packer Avenue
Crescent Blvd

# Identification

We see this:

A: XYXYXX

B: YYYXXY

Next we see:

A: XXXXXX

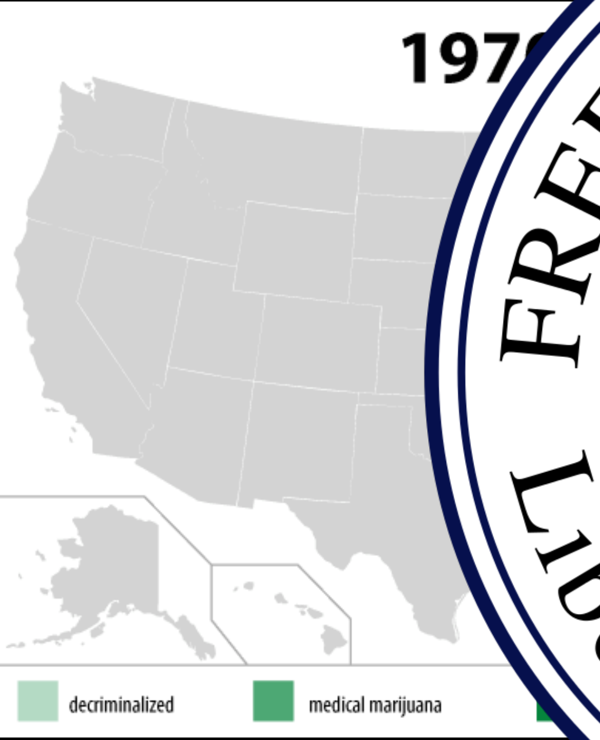
B: YYYYYY

Could be peer effects?

Could be homophily?

Yes, yes, …

And to know which one, we need "dynamic data"




FREE STATE PROJECT

FSP

Liberty in Our Lifetime

# Preference

Aggregation

# Preferences

What are your preference orderings over the colors: Red, green, blue?

How many preference orderings exist?

How many transitive preference orderings exist?

# Preference aggregation

Method: Pairwise vote (write them at the board)

| Individual | Ordering |
|---|---|
| Laura | R > G > B |
| Alexander | G > B > R |
| Ernesto | B > R > G |

# Preference aggregation

| Vote | Result |
|---|---|
| Green vs. Red | L & E: Red, A: Green |
| Blue vs. Green | A & L: Green, E: Blue |
| Red vs. Blue | E & A: Blue, L: Red |

Red > Green > Blue > Red

Intransitive!!

# Preference aggregation

Condorcet paradox!

# Epidemiology

Diffusion, SIS and networks

# Diffusion model

Let

- N : Number of individuals in population
- $X_t$ : Number of individuals with IT
- □ : Transmission rate
- c : Contact rate

# Diffusion model

Number of meetings in each time period:

$Nc$

Probability of contagion at t+1:

$(X_t/N)[(X_t-1)/N]\square$

Number of infected at t+1, i.e. $X_{t+1}$:

$X_{t+1} = X_t + Nc\square(X_t/N)[(X - 1)/N]$

# Diffusion models

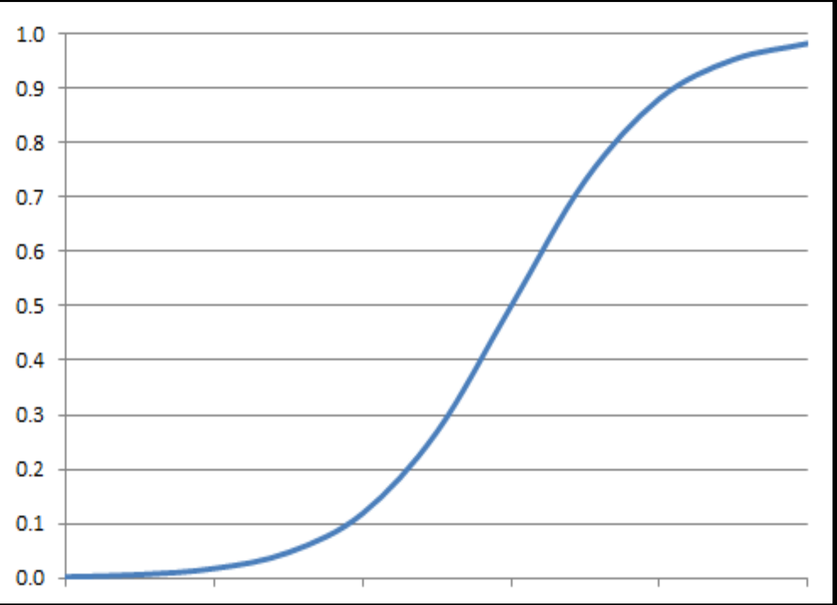
$(X_t/N)[(N - X_t)/N]$

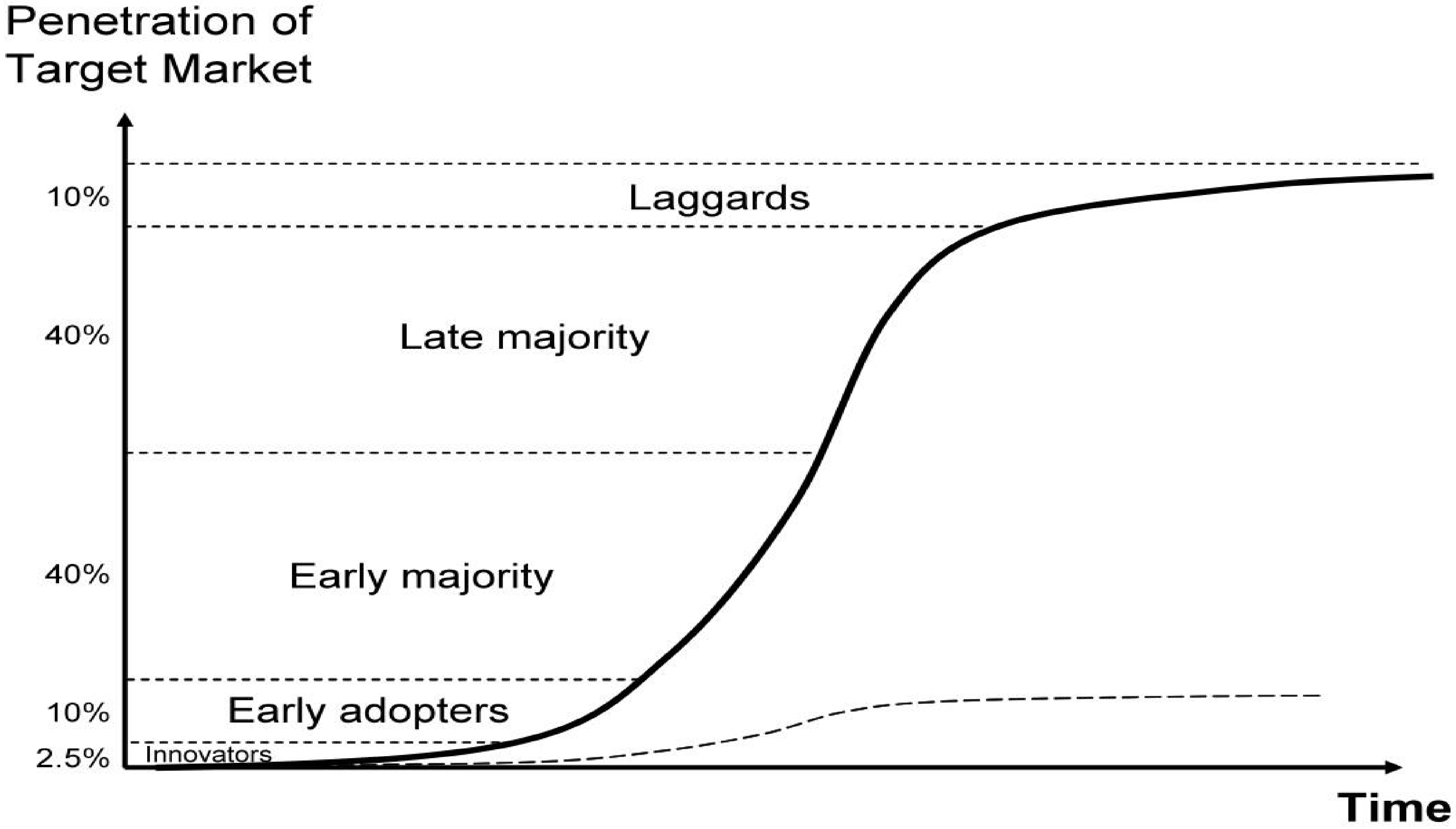
$(1/N)[(N-1)/N] \sim 1/N$

$[(N/2)/N][(N/2)/N] =$

$= ¼$

No tipping point




Penetration of
Target Market
10%
Laggards
40%
Late majority
40%
Early majority
10%
Early adopters
2.5%
Innovators
Time

# SIS - model

Susceptible Infectives Susceptible-model

a: rate of recovery

$$
\begin{aligned}
X_{t+1} &= X_t + Nc\square(X_t/N)X_t/N[(N-X_t)/N] - aX_t \\
&= X_t(1 + (c\square[(N-X_t)/N] - a)
\end{aligned}
$$

# SIS - model, continued

$X_t$ small : $\quad X_{t+1} \sim X_t(1 + (c\square - a))$

thus grows if $c\square - a > 0$, or equally if $c\square/a > 1$.

Let

$R_0$ : Basic reproduction number

$R_0 = c\square/a$

Tipping point.

# SIS, Diseases

| Disease | Basic reproduction number |
|---|---|
| Measles | 15 |
| Mumps | 5 |
| Flu | 3 |

# SIS, vaccine

Let

$v$ : ratio vaccinated

$r_0$ : post-vaccination reproduction number

$r_0 = R_0(1 - v)$

Thus vaccines help if,

$R_0(1 - v) \leq 1$, iff $v \geq 1 - 1/R_0$

Going back to the diseases, we then got!

# Simulations on networks

Networks -> Virus on a network

Biology -> Virus

Biology -> AIDS

What model is better?

# Markov models

In transition

# Markov models

On the board:

2-state example, p = .8, q=.2

Roll the numbers, with 1 and 0

Roll the numbers using matrix, [1 0] and [0 1]

Calculate equilibria

Do example with Single, Dating and In relationship, and solve.

# Markov Convergence Criteria

1. Finite number of states
2. Fixed transition probabilities
3. Any state can be reached (indirectly) from all states
4. Not a simple cycle

# For a Markov Process

Initial conditions doesn’t matter
History doesn’t matter
Intervening and changing state, it doesn’t matter

We will end up at equilibrium.

# Simulations on networks

Networks -> Virus on a network

Biology -> Virus

Biology -> AIDS

What model is better?

# The end.

Tack så mycket!